Numeric Response Questions

Ellipse

Q.1 Find the number of real tangents that can be drawn to the ellipse $3x^2 + 5y^2 = 32$ passing through (3,5).

Q.2 A man running round a race course notes that the sum of the distances of two flag-posts from him is always 10 metres and the distance between the flag-posts is 8 meters. The area of the path he encloses is $k\pi$ then find k.

Q.3 An ellipse having foci at (3,1) and (1,1) passes through the point (1,3). Then find its eccentricity.

Q.4 If the ecentricity of an ellipse be $5/8$ and the distance between its foci be 10, then find its latus rectum.

Q.5 Suppose S and S' are foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. If P is variable point on the ellipse and, if Δ is area of the triangle PSS", then find the maximum value of Δ

Q.6 Tangent at a point P on $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the x-axis at A and y-axis at B. The locus of the midpoint of AB is $\frac{a^2}{x^2} + \frac{b^2}{y^2} = k$, then find k.

Q.7 Find the radius of the circle passing through the foci of ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having its centre (0, 3).

Q.8 S_1, S_2 are foci of an ellipse of major axis of length 10 units and P is any point on the ellipse such that perimeter of $\triangle PS_1 S_2$ is 15. Find the eccentricity of ellipse.

Q.9 If extremities of diameter of the circle $x^2 + y^2 = 16$ are foci of an ellipse, then find the eocentricity of the ellipse, if its size is just sufficient to contain the circle.

Q.10 If B and B' are the ends of minor axis and S and S' are the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, then find the area of rhombus SBS'B' formed.

Q.11 If the eccentricity of the two ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are equal, then find the value of $\frac{a}{b}$.

Q.12 For a point P on the ellipse $9x^2 + 36y^2 = 324$, with foci S and S, find value of SP + S 'P.

Q.13 Find the number of values of c such that the straight line $y = 4x + c$ touches the curve $x^2/4 + y^2 = 1$.

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Q.14 If the eccentricity of the ellipse $4x^2 + 9y^2 - 8x - 6y + 1 = 0$ is $\frac{\sqrt{5}}{k}$ then find k.

Q.15 Find the product of the of length perpendiculars drawn from the foci upon any tangent to the ellipse $3x^2 + 4y^2 = 12$.

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ANSWER KEY

1. 2.00	**2.** 15.00	**3.** 0.41	**4.** 9.75	**5.** 12.00	**6.** 4.00	**7.** 4.00
8. 0.50	**9.** 0.70	**10.** 24.00	**11.** 2.60	**12.** 12.00	**13.** 2.00	**14.** 3.00
15. 3.00						

Hints & Solutions

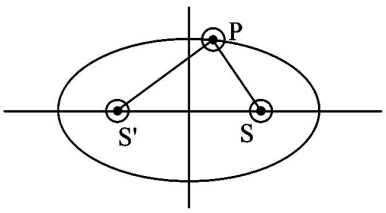
1. $S = 3x^2 + 5y^2 - 32 = 0$

$(3, 5)$

$S_1 = 27 + 5 \times 25 - 32 > 0$

Two tangents

2.



$PS + PS' = 10$

$2a = 10$

$a = 5$

$2ae = 8$

$ae = 4$

$e = 4/5$

$b^2 = a^2 (1 - e^2)$

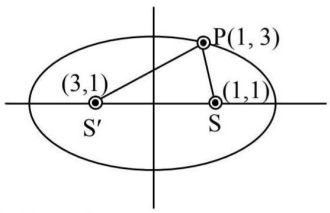
$b^2 = 25\left(1 - \frac{16}{25}\right)$

$b = 3$

$A = \pi ab = \pi \times 5 \times 3$

$= 15\pi$

3.



$PS + PS' = 2a$

$\sqrt{4+4} + 2 = 2a$

$2(\sqrt{2}+1) = 2a \Rightarrow a = \sqrt{2}+1$

$2ae = \sqrt{4+0} \Rightarrow 2ae = 2 \Rightarrow ae = 1$

$e = \frac{1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$e = \sqrt{2} - 1$

4. Given : $e = \frac{5}{8}$

and $2ae = 10$

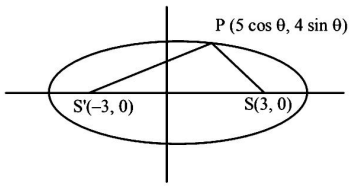
$\Rightarrow ae = \frac{10}{2} = 5 \Rightarrow a = \frac{5}{e} = \frac{5}{\frac{5}{8}} = 8$

Now $b^2 = a^2 (1 - e^2)$

$b^2 = 64\left(1 - \frac{25}{64}\right) = 39$

$\therefore$ Length of L.R. $= \frac{2b^2}{a} = \frac{2(39)}{8} = \frac{39}{4}$

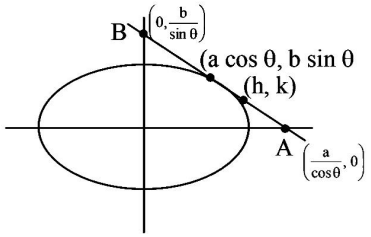
5.



Area $= \frac{1}{2} \times 6 \times 4 \sin\theta = 12 \sin\theta$

$(\text{Area})_{max} = 12$

6.



$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$

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$$h = \frac{\frac{a}{\cos\theta} + 0}{2}, \quad k = \frac{0 + \frac{b}{\sin\theta}}{2}$$

$$\cos\theta = \frac{a}{2h}, \quad \sin\theta = \frac{b}{2k}$$

$$\cos^2\theta + \sin^2\theta = 1$$

$$\frac{a^2}{4h^2} + \frac{b^2}{4k^2} = 1$$

$$\therefore \text{ locus } \frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$$

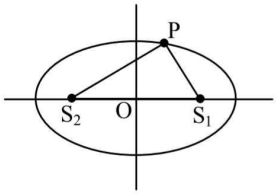
7. $\frac{x^2}{16} + \frac{y^2}{9} = 1$

$$e = \sqrt{1 - \frac{9}{16}} = \frac{\sqrt{7}}{4}$$

foci are $(\pm\sqrt{7}, 0)$

Radius of circle $= \sqrt{7+9} = 4$

8.



$2a = 10$

$\Rightarrow a = 5$

$PS_1 + PS_2 + S_1S_2 = 15$

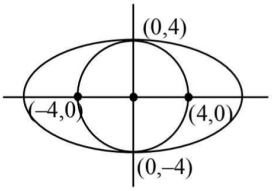
$\Rightarrow 2a + 2ae = 15$

$\Rightarrow 10 + 10e = 15$

$10e = 5$

$\Rightarrow e = \frac{5}{10}$

9.



$2ae = 8$

$b = 4$

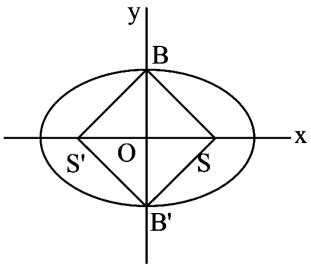
$b^2 = a^2 - a^2e^2$

$16 = a^2 - 16$

$a^2 = 32$

$$e = \sqrt{1 - \frac{16}{32}} = \frac{1}{\sqrt{2}}$$

10.



$$\text{Area} = 4.\ \frac{1}{2}(ae)(b)$$

$$= 2abe$$

$$= 2.(5.)(3)\sqrt{1 - \frac{9}{25}}$$

$$= 30.\ \frac{4}{5} = 24$$

11. $\sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{25}{169}}$

$$\Rightarrow \frac{a}{b} = \frac{13}{5}$$

12. $\frac{x^2}{36} + \frac{y^2}{9} = 1$

$\therefore SP + S'P = 2a = 12$

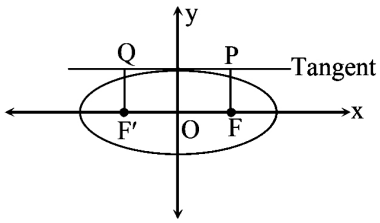
13. $c = \pm\sqrt{a^2m^2 + b^2}$

$c = \pm\sqrt{4(16)+1} = \pm\sqrt{65}$ (two values)

14. $e = \sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

15. Ellipse $3x^2 + 4y^2 = 12$

$$\Rightarrow \frac{x^2}{4} + \frac{y^2}{3} = 1$$



$(PF)\ (QF') = (b)\ (b) = b^2 = 3$

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